Electrical Technology

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AC Circuits

- An AC circuit consists of a combination of circuit elements and a power source
- The power source provides an alternative voltage, Δv
- Notation Note
  - Lower case symbols will indicate instantaneous values
  - Capital letters will indicate fixed values

AC Voltage

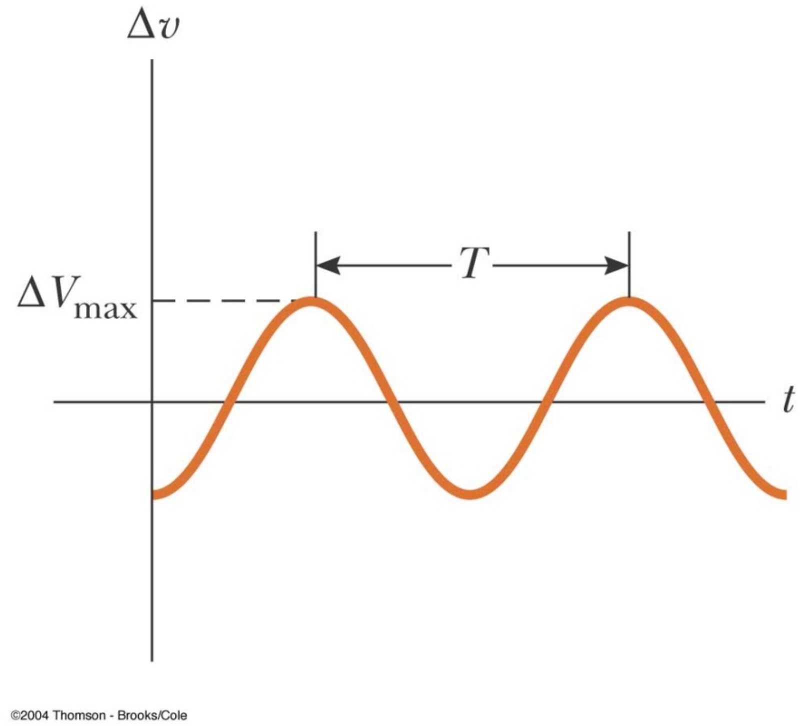
- The output of an AC power source is sinusoidal and varies with time according to the following equation:
  - $\Delta v = \Delta V_{max} \sin \omega t$
    - Δv is the instantaneous voltage
    - ΔV_{max} is the maximum output voltage of the source
    - ω is the angular frequency of the AC voltage

AC Voltage, cont.

- The angle frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

  - f is the frequency of the source
  - T is the period of the source
- The voltage is positive during one half of the cycle and negative during the other half

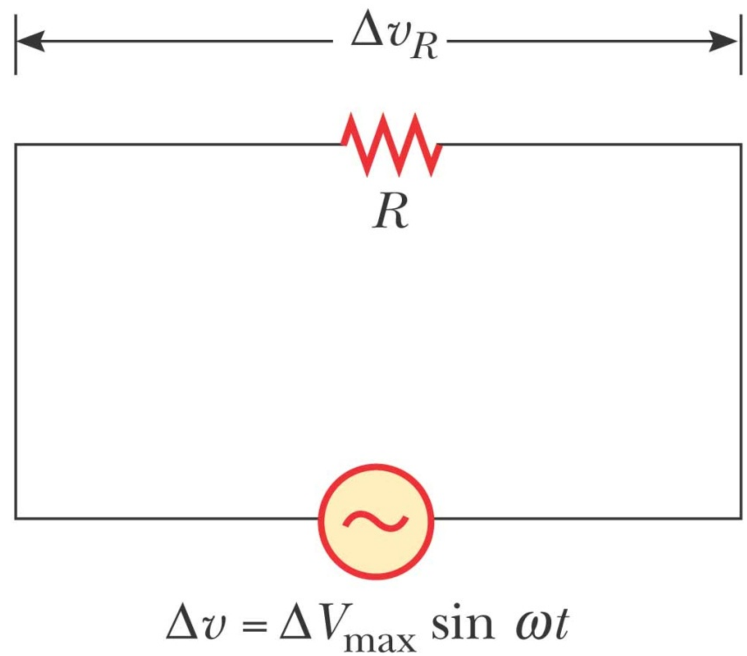


AC Voltage, final

- The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time
- Commercial electric power plants in the US use a frequency of 60 Hz
  - This corresponds with an angular frequency of 377 rad/s

Resistors in an AC Circuit

- Consider a circuit consisting of an AC source and a resistor
- The AC source is symbolized by 
- $\Delta v = \Delta v_R = \Delta v_{max} \sin \omega t$
- Δv_R is the instantaneous voltage across the resistor



Resistors in an AC Circuit, 2

- The instantaneous current in the resistor is

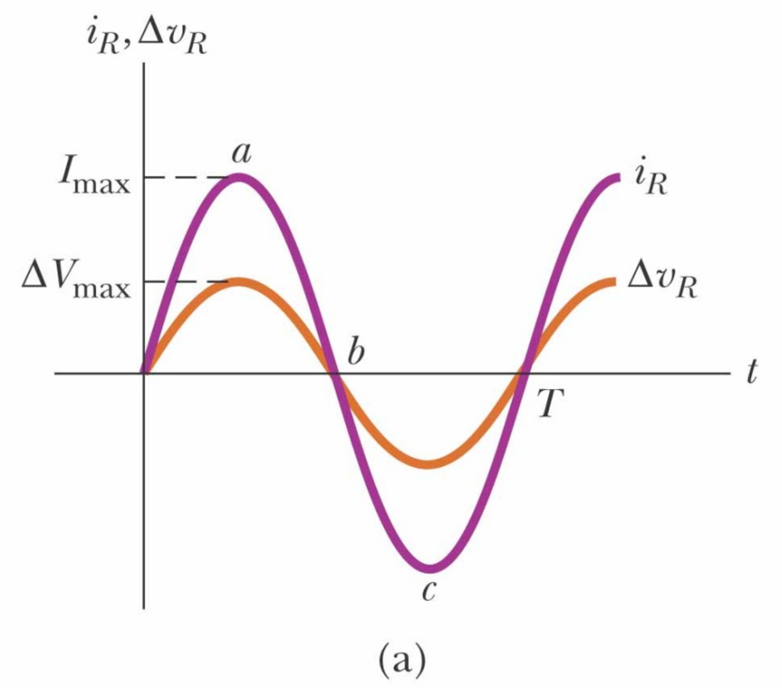
$$i_R = \frac{\Delta v_R}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t$$

- The instantaneous voltage across the resistor is also given as

$$\Delta v_R = I_{max} R \sin \omega t$$

Resistors in an AC Circuit, 3

- The graph shows the current through and the voltage across the resistor
- The current and the voltage reach their maximum values at the same time
- The current and the voltage are said to be *in phase*

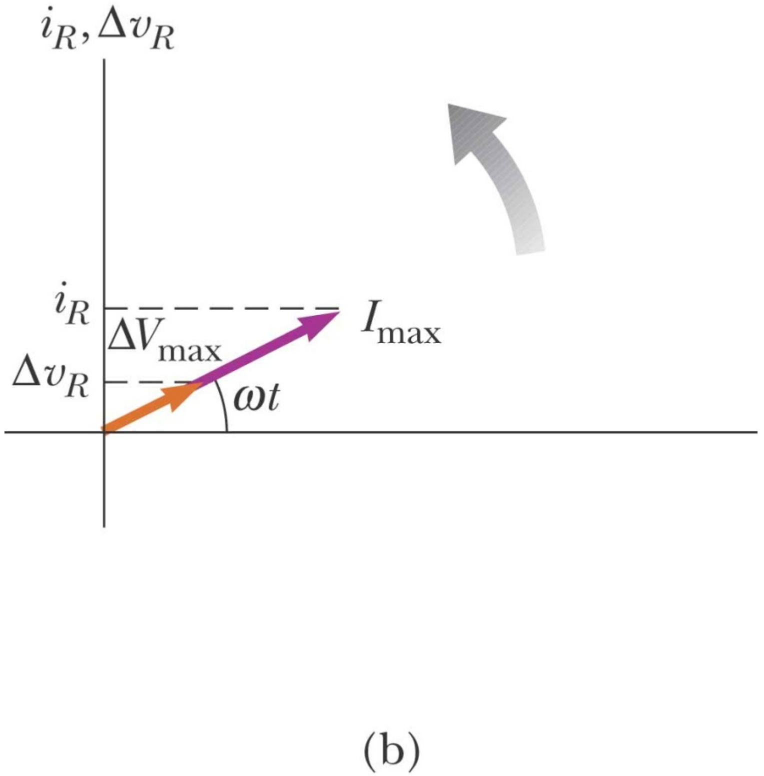


Resistors in an AC Circuit, 4

- For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor
- The direction of the current has no effect on the behavior of the resistor
- Resistors behave essentially the same way in both DC and AC circuits

Phasor Diagram

- To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used
- A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents



Phasors, cont.

- The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable
- The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents

rms Current and Voltage

- The average current in one cycle is zero
- The *rms current* is the average of importance in an AC circuit
  - rms stands for *root mean square*

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

- Alternating voltages can also be discussed in terms of rms values

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707\, \Delta V_{max}$$

Power

- The rate at which electrical energy is dissipated in the circuit is given by
  - $P = i^2 R$
    - where i is the *instantaneous current*
    - the heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value
    - The maximum current occurs for a small amount of time

Power, cont.

- The average power delivered to a resistor that carries an alternating current is

$$P_{av} = I_{rms}^2 R$$

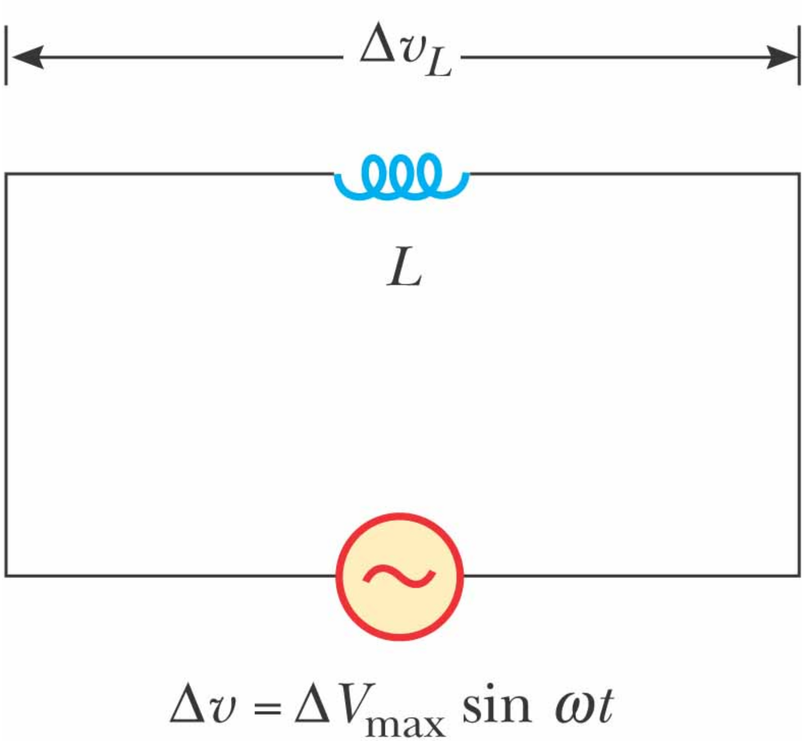
Inductors in an AC Circuit

- Kirchhoff's loop rule can be applied and gives:

$$\Delta v + \Delta v_L = 0, \text{ or}$$

$$\Delta v - L\frac{di}{dt} = 0$$

$$\Delta v = L\frac{di}{dt} = \Delta V_{max}\sin\omega t$$



Current in an Inductor

- The equation obtained from Kirchhoff's loop rule can be solved for the current

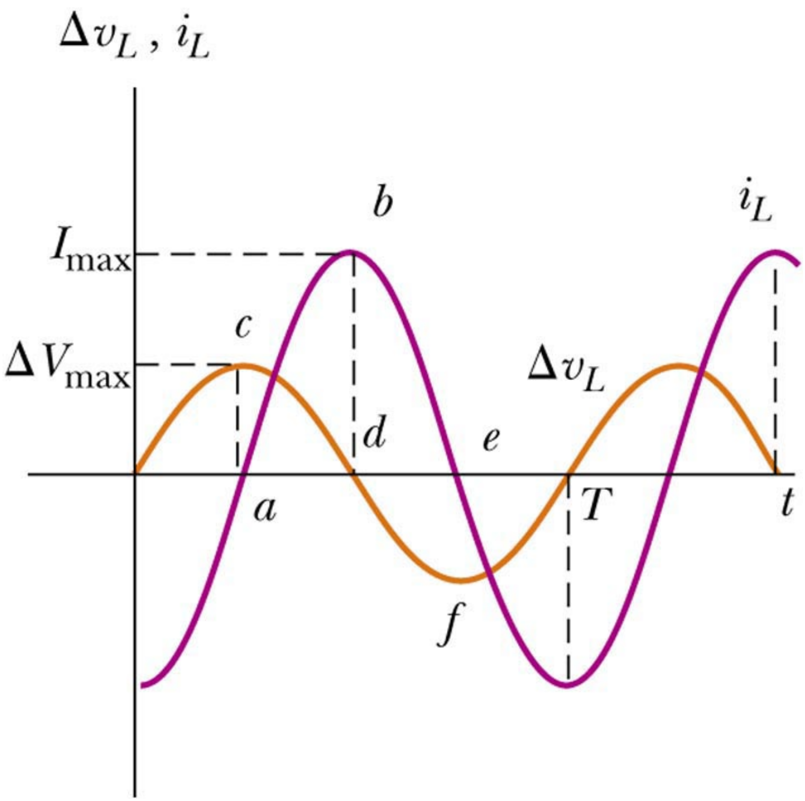
$$i_L = \frac{\Delta V_{max}}{L}\int \sin \omega t \; dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t$$

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_{max} = \frac{\Delta V_{max}}{\omega L}$$

- This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are *out* of phase by (π/2) rad = 90°

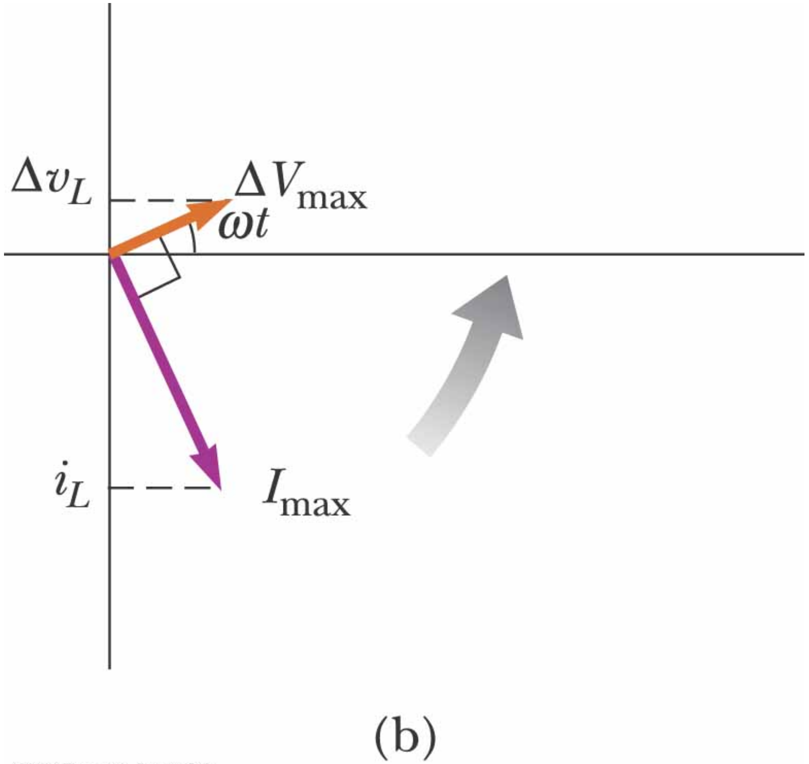
Phase Relationship of Inductors in an AC Circuit

- The current in the circuit is impeded by the back emf of the inductor
- For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90° ($\pi/2$)



Phasor Diagram for an Inductor

- The phasors are at 90° with respect to each other
- This represents the phase difference between the current and voltage
- Specifically, the current lags behind the voltage by 90°



Inductive Reactance

- The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance
- Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies
- The factor is the **inductive reactance** and is given by:
  - $X_L = \omega L$

Inductive Reactance, cont.

- Current can be expressed in terms of the inductive reactance

$$I_{max} = \frac{\Delta V_{max}}{X_L}$$

- As the frequency increases, the inductive reactance increases
  - This is consistent with Faraday's Law:
    - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the inductance and a decrease in the current

Voltage Across the Inductor

- The instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di}{dt}$$
$$= -\Delta V_{\max} \sin \omega t$$
$$= -I_{\max} X_L \sin \omega t$$

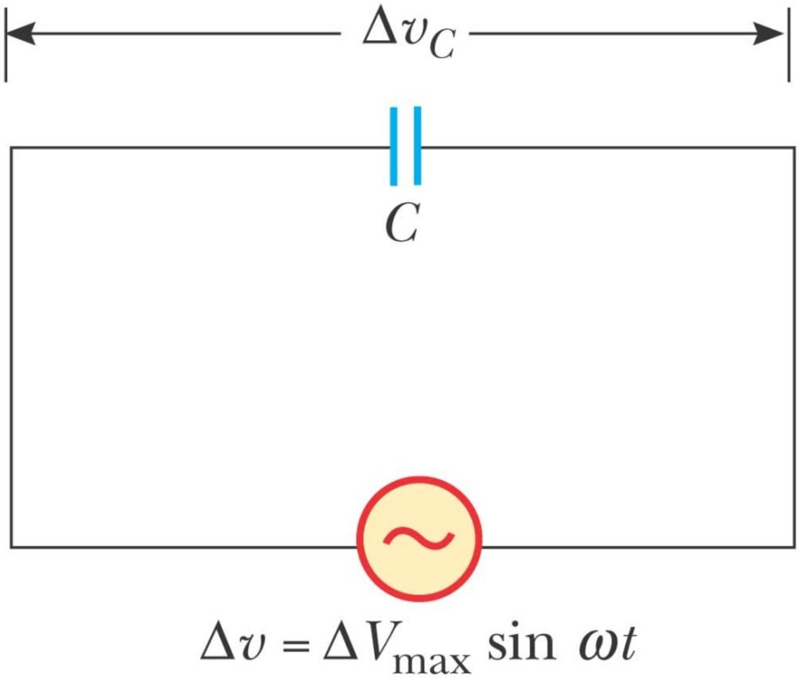
Capacitors in an AC Circuit

- The circuit contains a capacitor and an AC source
- Kirchhoff's loop rule gives:

  $\Delta v + \Delta v_c = 0$ and so

  $\Delta v = \Delta v_C = \Delta V_{max} \sin \omega t$

  - Δv_c is the instantaneous voltage across the capacitor



Capacitors in an AC Circuit, cont.

- The charge is $q = C\,\Delta V_{max} \sin \omega t$
- The instantaneous current is given by

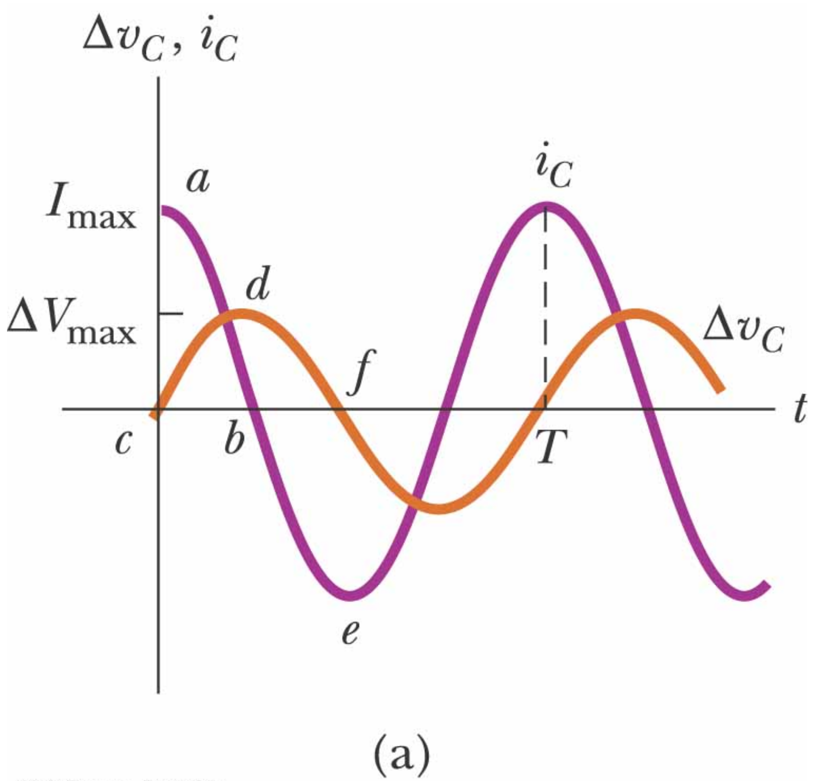
$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t$$

$$\text{or } i_C = \omega C \Delta V_{max} \sin\left(\omega t + \frac{\pi}{2}\right)$$

- The current is $\pi/2$ rad = 90° out of phase with the voltage

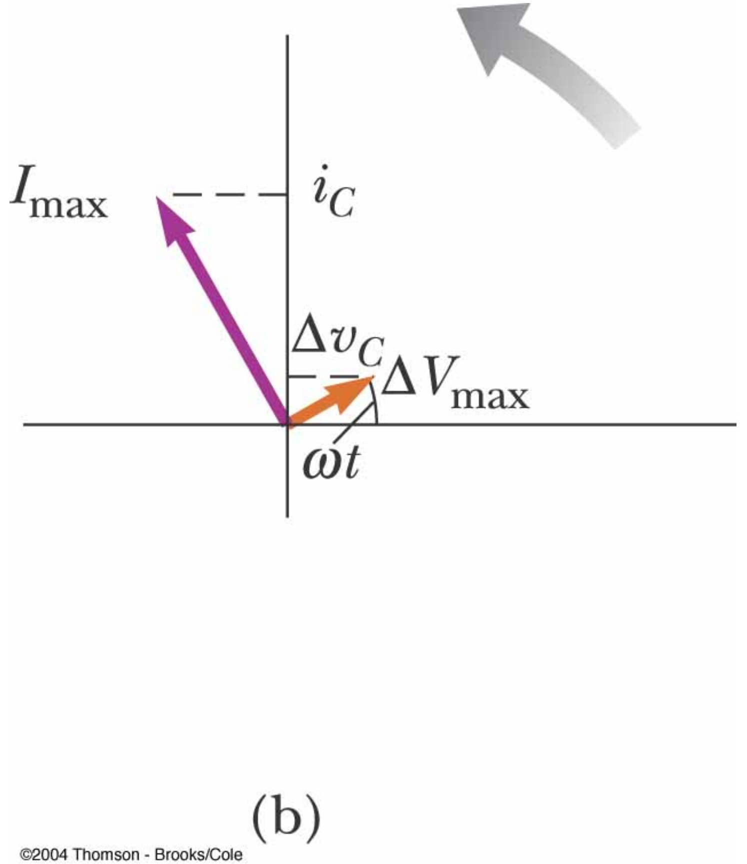
More About Capacitors in an AC Circuit

- The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value
- The voltage lags behind the current by 90°



Phasor Diagram for Capacitor

- The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by 90º
  - This is equivalent to saying the voltage lags the current



Capacitive Reactance

- The maximum current in the circuit occurs at cos ωt = 1 which gives

$$I_{max} = \omega C \Delta V_{max} = \frac{\Delta V_{max}}{(1/\omega C)}$$

- The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \quad \text{and} \quad I_{max} = \frac{\Delta V_{max}}{X_C}$$

Voltage Across a Capacitor

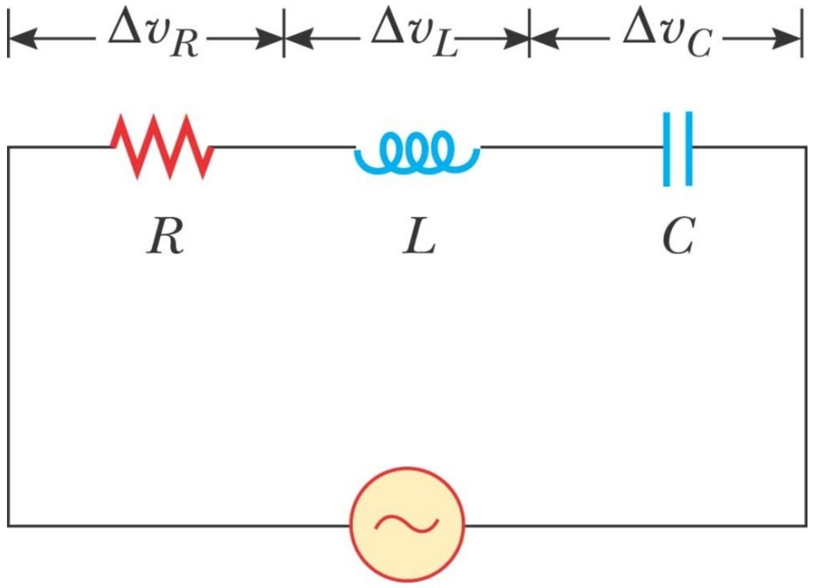
- The instantaneous voltage across the capacitor can be written as

  $\Delta v_C = \Delta V_{max} \sin \omega t = I_{max} X_C \sin \omega t$

- As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases
- As the frequency approaches zero, X_C approaches infinity and the current approaches zero
  - This would act like a DC voltage and the capacitor would act as an open circuit

The *RLC* Series Circuit

- The resistor, inductor, and capacitor can be combined in a circuit
- The current in the circuit is the same at any time and varies sinusoidally with time



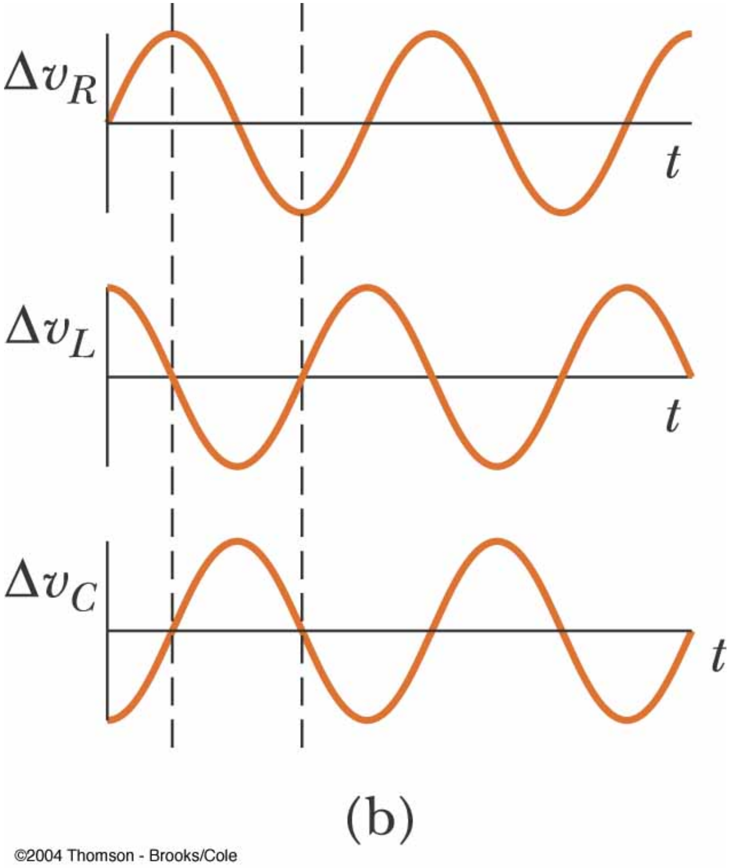
(a)

The *RLC* Series Circuit, cont.

- The instantaneous voltage would be given by $\Delta v = \Delta V_{max} \sin \omega t$
- The instantaneous current would be given by $i = I_{max} \sin (\omega t - \varphi)$
  - φ is the *phase angle* between the current and the applied voltage
- Since the elements are in series, the current at all points in the circuit has the same amplitude and phase

i and *v* Phase Relationships – Graphical View

- The instantaneous voltage across the resistor is in phase with the current
- The instantaneous voltage across the inductor leads the current by 90°
- The instantaneous voltage across the capacitor lags the current by 90°



i and *v* Phase Relationships – Equations

- The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin\left(\omega t + \frac{\pi}{2}\right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin\left(\omega t - \frac{\pi}{2}\right) = -\Delta V_C \cos \omega t$$

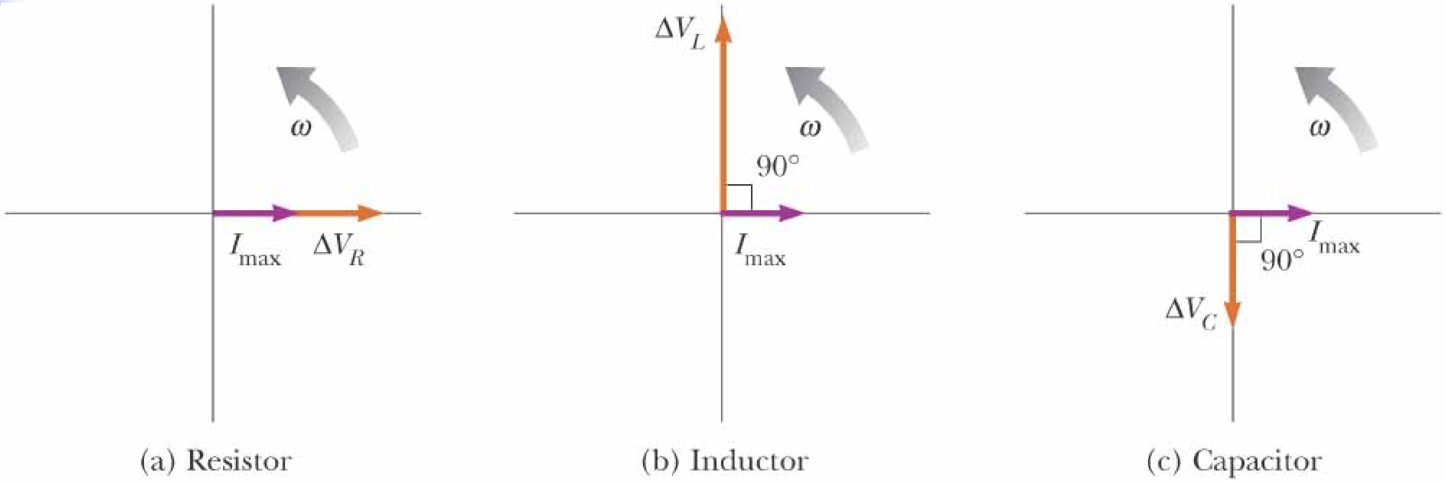
More About Voltage in *RLC* Circuits

- ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{\max} R$
- ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{\max} X_L$
- ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{\max} X_C$
- In series, voltages add and the instantaneous voltage across all three elements would be

  $$\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$$

  - Easier to use the phasor diagrams

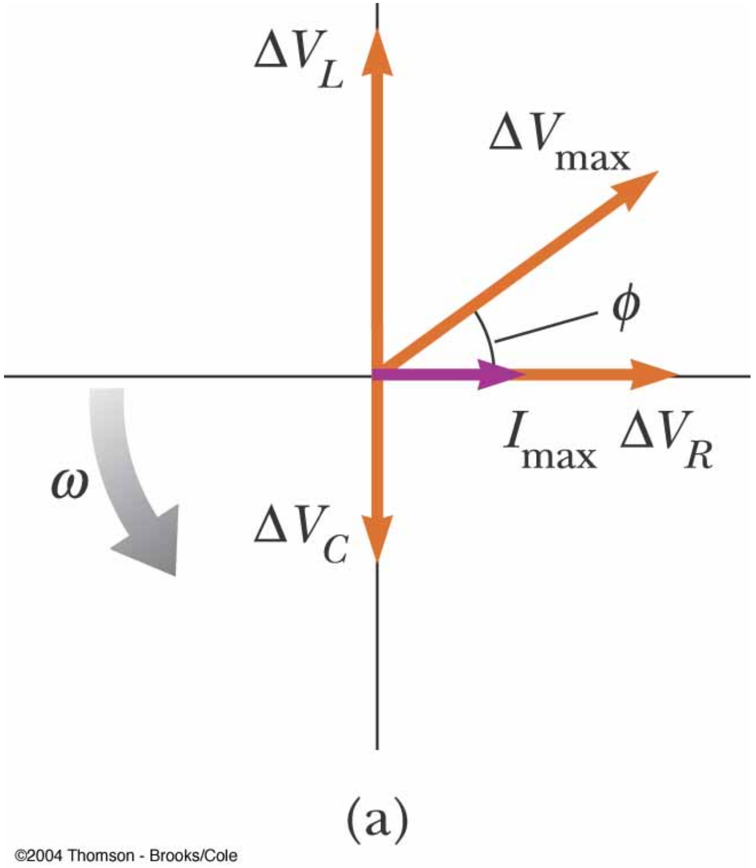
Phasor Diagrams



- To account for the different phases of the voltage drops, vector techniques are used
- Remember the phasors are rotating vectors
- The phasors for the individual elements are shown

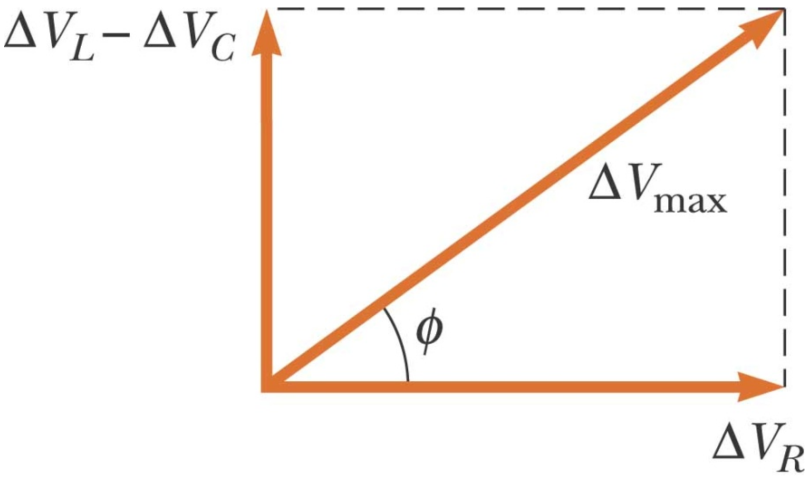
Resulting Phasor Diagram

- The individual phasor diagrams can be combined
- Here a single phasor I_{max} is used to represent the current in each element
  - In series, the current is the same in each element



Vector Addition of the Phasor Diagram

- Vector addition is used to combine the voltage phasors
- ΔV_L and ΔV_C are in opposite directions, so they can be combined
- Their resultant is perpendicular to ΔV_R



(b)

Total Voltage in *RLC* Circuits

- From the vector diagram, ΔV_{max} can be calculated

$$\Delta V_{max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2}$$

$$= \sqrt{(I_{max} R)^2 + \left(I_{max} X_L - I_{max} X_C\right)^2}$$

$$\Delta V_{max} = I_{max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Impedance

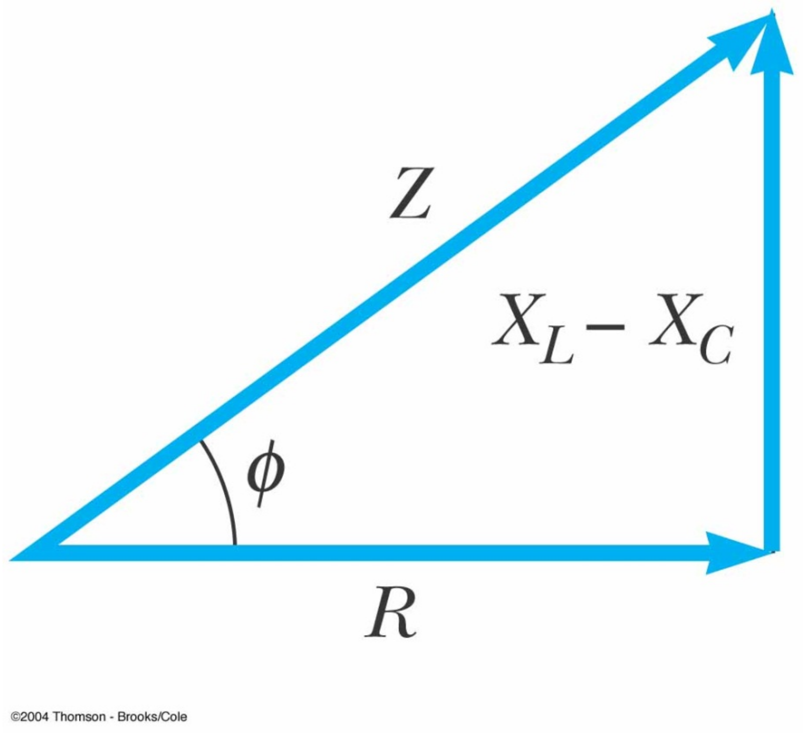
- The current in an RLC circuit is

$$I_{max} = \frac{\Delta V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\Delta V_{max}}{Z}$$

- *Z* is called the impedance of the circuit and it plays the role of resistance in the circuit, where $Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$
  - Impedance has units of ohms
  - Also, $\Delta V_{max} = I_{max} Z$

Impedance Triangle

- Since I_{max} is the same for each element, it can be removed from each term in the phasor diagram
- The result is an impedance triangle



Impedance Triangle, cont.

- The impedance triangle confirms that

$$Z \equiv \sqrt{R^2 + (X_L - X_C)^2}$$

- The impedance triangle can also be used to find the phase angle, φ

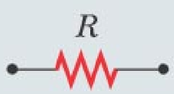
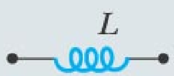
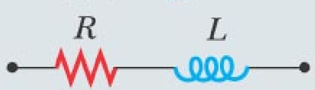
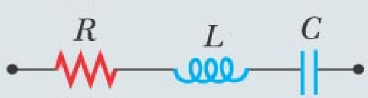
$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- The phase angle can be positive or negative and determines the nature of the circuit
- Also, $\cos \varphi = \dfrac{R}{Z}$

Summary of Circuit Elements, Impedance and Phase Angles

Table 33.1

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the elements.

Power in an AC Circuit

- The average power delivered by the generator is converted to internal energy in the resistor
  - $P_{av} = 1/2\, I_{max} \Delta V_{max} \cos \varphi = I_{rms} \Delta V_{rms} \cos \varphi$
  - $\cos \varphi$ is called the *power factor* of the circuit
- We can also find the average power in terms of *R*
  - $P_{av} = I^2_{rms} R$

Power in an AC Circuit, cont.

- The average power delivered by the source is converted to internal energy in the resistor
- No power losses are associated with pure capacitors and pure inductors in an AC circuit
  - In a capacitor, during one-half of a cycle energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor
  - In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit

Power and Phase

- The power delivered by an AC circuit depends on the phase
- Some applications include using capacitors to shift the phase in heavy motors so that excessively high voltages are not needed